A Review of the Application of Multiobjective Evolutionary Fuzzy Systems: Current Status and Further Directions

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***Abstract*—Over the past few decades, fuzzy systems have been widely used in several application fields, thanks to their ability to model complex systems. The design of fuzzy systems has been successfully performed by applying evolutionary and, in particular, genetic algorithms, and recently, this approach has been extended by using multiobjective evolutionary algorithms, which can consider multiple conflicting objectives, instead of a single one. The hybridization between multiobjective evolutionary algorithms and fuzzy systems is currently known as multiobjective evolutionary fuzzy systems. This paper presents an overview of multiobjective evolutionary fuzzy systems, describing the main contributions on this field and providing a two-level taxonomy of the existing proposals, in order to outline a well-established framework that could help researchers who work on significant further developments. Finally, some considerations of recent trends and potential research directions are presented.**

***Index Terms*—Accuracy–interpretability tradeoff, fuzzy association rule mining, fuzzy control, fuzzy rule-based systems (FRBSs), multiobjective evolutionary algorithms (EAs), multiobjective evolutionary fuzzy systems (MOEFSs).**

I. Introduction

FUZZY systems can be considered universal approximators, i.e., they can approximate any real continuous function in a compact set to arbitrary accuracy [1]. They have been widely used in several application fields, such as control [2]–[4], classification [5]–[8], regression [9], [10], and general data mining problems, due to their ability to handle imprecision and uncertainty and to describe the behavior of complex systems without requiring a precise mathematical model.

The most common fuzzy models consist of a collection of logical fuzzy rules and are known as fuzzy rule-based systems (FRBSs). They can be roughly divided into several families, depending on the type of fuzzy sets used in the rule.

The two most popular FRBSs are linguistic fuzzy models, also called Mamdani-type [4] and Takagi–Sugeno–Kang (TSK) fuzzy models [11]. The difference between these models lies in the consequent of their fuzzy rules, which is an output action or a class in the first case and a polynomial function in the second. Another category of fuzzy models is represented by scatter partition-based FRBSs [12], which differ from linguistic FRBSs as their rules are semantic-free.

The automatic design of FRBSs can be considered an optimization task or a search problem. Evolutionary algorithms (EAs) and genetic algorithms (GAs) [13], [14] have been employed to carry this out, thanks to their ability to deal with large search spaces and to find near-optimal solutions without a precise description of the problem. Moreover, they can easily incorporate *a priori* knowledge into the model. The hybridization of fuzzy systems and GAs is known as genetic fuzzy systems (GFSs) [15], [16], and can nowadays be considered a mature area.

The flexibility of fuzzy systems makes them applicable to a wide range of problems. From among them, problems with multiple conflicting objectives are of particular interest to researchers, as they are very common and arise wherever optimal decisions need to be taken. In these kinds of problems, the improvement of an objective leads to the deterioration of the others; therefore, there is usually no single solution that simultaneously optimizes all objectives.

This problem is tackled using multiobjective evolutionary algorithms (MOEAs) [17], [18] for the design of fuzzy systems. Multiobjective optimization leads to a set of fuzzy models with different tradeoffs between objectives instead of a single, usually biased fuzzy model. Afterward, a decision maker (or an automatic decision making process) can select, depending on his/her requirements, the most suitable model. These hybrid approaches are known as multiobjective evolutionary fuzzy systems (MOEFSs).

In this paper, we analyze the state of the art of MOEFSs, by describing a collection of proposals that focus on this topic. Intending to show a well-established framework, we present a two-level taxonomy to classify contributions, in which the first level is based on the multiobjective nature of the problem faced, i.e, the type of objectives used, and the second level is based on the type of FRBS components optimized during the evolutionary process. Both of them determine the search space type

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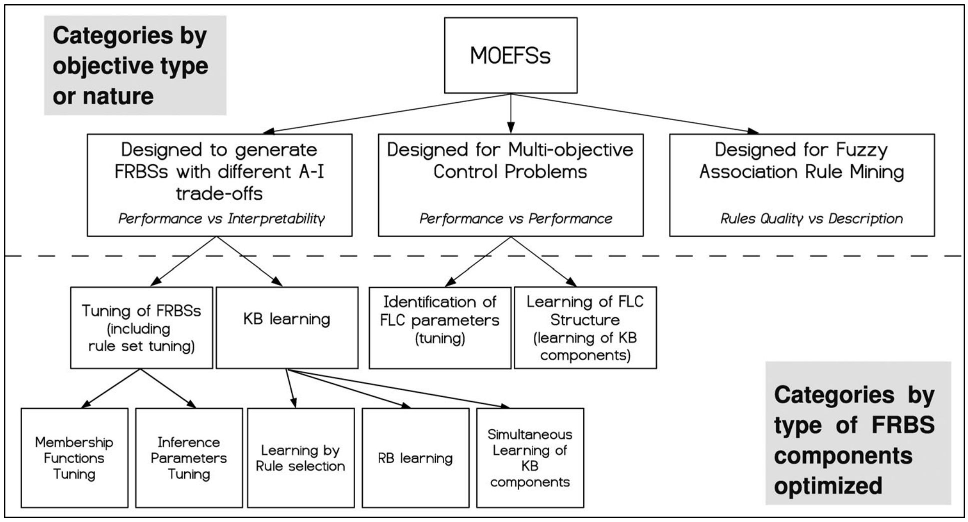

Fig. 1. Two-level taxonomy based on the type of the objectives optimized (1st level) and on the type of GFS used (second level).

and complexity, which involve different considerations when applying MOEFSs.

This way, this taxonomy could help researchers to easily find existing proposals that are related to a particular branch and to focus on significant further developments. Finally, we discuss some current trends and prospects.

To keep this study up-to-date, improving its visibility and providing additional materials, we have developed an associated web page that can be found at http://sci2s.ugr.es/moefs-review/. This web page includes some basic preliminary concepts that are related to the topic of MOEFSs: introduction to FRBSs, introduction to evolutionary multiobjective optimization (EMOO), and a definition of MOEFSs as the application of EMOO to FRBS derivation. It also presents the proposed taxonomy and shows a summary of the state of the art, grouping the studied contributions in the form of tables and including the links to each paper digital object identifier (DOI).

This paper is organized as follows. Section II introduces a two-level taxonomy of proposals: first based on the multiobjective nature of the problem tackled and second based on the type of FRBS components optimized. The following sections contain the descriptions of the main studies that are related to each field. Section III groups works that deal with MOEFSs applied to the accuracy–interpretability tradeoff of FRBSs. Section IV describes MOEFSs that are applied to multiobjective control problems. Section V focuses on studies that apply MOEFSs to mine fuzzy association rules. In Section VI, some new trends and further developments are discussed. Finally, some conclusions are drawn in Section VII.

II. Taxonomy Based on the Application of Multiobjective Evolutionary Fuzzy Systems

In this paper, we take into consideration a collection of papers in which MOEFSs are applied to different problem domains. Because of the large number of contributions to the field, we propose a two-level taxonomy, which is shown in Fig. 1, in order to jointly analyze the different types of MOEFSs. The first level gathers contributions depending on the multiobjective nature of the handled problem, i.e., the type of the objectives optimized. The second one groups papers depending on the type of FRBS components optimized during the evolutionary process. In fact, both of them affect the type and the complexity of the search space, and therefore the way in which MOEFSs are applied.

This way, the first main category includes contributions in which MOEFSs are designed to generate FRBSs with different tradeoffs between accuracy and interpretability. In this case, at least one of the objectives is always related to the interpretability of the obtained model, regardless of the problem considered. A considerable number of papers can be found in this group, since interpretability is one of the most important aspects of FRBSs. While the accuracy is difficult to improve, interpretability is easy to obtain, since interpretable models can even be provided by hand. These differences between both types of objectives influence the optimization process.

The second main category gathers contributions in which MOEFSs are applied to multiobjective control problems. The considered objectives strictly depend on the particular kind of problem that is taken into account, and usually all of them are related to performance issues of the control system. Therefore, the tradeoff and the search space will be different for each problem and dependent on the problem itself.

The third main category groups contributions in which MOEFSs are applied to fuzzy association rule mining. The aim of rule mining is to find a set of fuzzy association rules that reliably represents the knowledge hidden in a database. In this case, the objectives are used to describe the quality of the obtained rules, i.e., their accuracy and interestingness. To this end, support and confidence are the major factors in measuring the quality of an association rule, although other metrics exist. The aim of the optimization process is not only to improve the general tradeoff between objectives for the whole set of rules, but also to obtain a large number of rules, each of them satisfying the objectives to different degrees.

This section illustrates the proposed taxonomy and includes the description of subcategories for each main category.

A. Multiobjective Evolutionary Fuzzy Systems Designed to Generate Fuzzy Rule-Based Systems With Different Accuracy–Interpretability Tradeoffs

One of the main uses of FRBSs is in the approximation of a real system with a fuzzy model, which can be used to explain, simulate, or predict the behavior of the original system. Of course, the higher the accuracy, the more reliable the model.

Initially, the interpretability of the obtained models was neglected, since single-objective EAs permit the optimization of only a single metric. The problem of improving accuracy while maintaining or even improving the interpretability of a fuzzy model was first faced in the mid-1990s by Ishibuchi and his group [19], and the comprehensibility of fuzzy models began to be integrated into the optimization process, thanks to the application of MOEAs to fuzzy systems.

Ever since, interpretability has acquired an increasing importance in the field of MOEFSs. Because of its subjectivity, the main problem is to find a shared definition of interpretability and to measure this characteristic in the obtained models, since several issues need to be taken into account to obtain a human-interpretable model.

Over the course of the past decade, several works have analyzed the interpretability problem in FRBSs [20], looking for interpretability measures that could be universally accepted by the research community [21]–[23]. This effort has continued in recent years, as demonstrated by the review papers presented in [24]–[27], which aim to propose a well-established framework to characterize and classify these measures.

Despite this, there are still no commonly accepted measures, and even the terms used in the area (comprehensibility, readability, completeness, consistency, etc.) are confusing and used as synonyms, even if they refer to different concepts. Nowadays, researchers agree on the need to consider two groups of interpretability measures:

1) complexity-based interpretability measures, which are used to decrease the complexity of the fuzzy model (number of rules, number of antecedents in a rule, etc.);
2) semantic-based interpretability measures, which are used to preserve the semantics associated with membership functions (distinguishability, coverage, etc.) and rules (consistency, etc.).

Classically, interpretability indices have only focused on the former group, when evaluating the overall interpretability of a fuzzy model. On the other hand, the definition of good semantic interpretability measures is still an open problem, since they are strongly affected by subjectivity. To this end, several indices have been proposed recently [26], [28], [29].

Considering the importance of the accuracy-interpretability tradeoff for the research community, this first category includes contributions in which MOEFSs are designed to handle this tradeoff that deals with this concept. Because of the huge number of existent works, we organized them into a second-level grouping, according to the taxonomy of GFSs presented in [16] (see Fig. 1), and thus considering the components of the FRBS that are managed by the optimization process (for further information on the types of FRBSs and knowledge base (KB) components, see the associated web page http://sci2s.ugr.es/moefs-review/).

1) Tuning of FRBS components, combined or not, with a rule set tuning process: A predefined KB is tuned by the optimization process, i.e., the parameters of the system (shape of membership functions in the data base (DB), inference parameters, etc.) are modified to obtain more accurate systems. In order to keep the system simple or to reduce complexity, in some cases a rule selection process, which is used as a postprocessing method, can be integrated in the optimization: From the initial rule base (RB), only necessary rules are selected. This approach can be considered a rule set tuning process. The contributions belonging to this category are further divided into two subcategories, named membership function tuning and inference parameter tuning.
2) KB learning: Papers belonging to this category consider the learning of the DB and/or RB. This group is further divided into three subcategories: learning by rule selection, RB learning, and simultaneous learning of KB components. In this case, the rule selection process is used to perform a learning of the RB.

The majority of works use a linguistic fuzzy model, since it is the most interpretable type of FRBS. However, there are a small number of works in which interpretability is considered even in a TSK-type FRBS. Because of their particularities, these contributions will be described at the end of this section.

B. Multiobjective Evolutionary Fuzzy Systems Designed for Multiobjective Control Problems

The performance of traditional controllers depends on their accuracy in modeling the system's dynamics. When designing a controller, the first problem appears if the processes are imprecisely described or are controlled by humans, without recourse to mathematical models, algorithms or a deep understanding of the physical processes involved. A further problem concerns how to design adaptive models, i.e., intelligent control systems that involve a learning or adaptation process when system parameters change.

Thus, it can be difficult to identify an accurate dynamic model to design a traditional controller. In these cases, fuzzy logic represents a powerful tool to deal with the problem of knowledge representation in an environment of uncertainty and imprecision. Furthermore, in control system design, there are often multiple objectives to be considered. These objectives are sometimes conflicting, causing an inevitable tradeoff among them, and no single design solution emerges as the best with respect to all objectives. These considerations have led to the application of MOEAs in the design of fuzzy logic controllers (FLCs).

The design of an FLC includes obtaining a structure for the controller and the corresponding numerical parameters. MOEAs can manage these problems by encoding both structure and parameters in one chromosome that represents the whole FLC. Therefore, in this second group, works will be explained considering the following two categories [30]:

1) identification of controller parameters and/or rules (e.g., tuning of membership function parameters, rule selection as a postprocessing method);
2) learning of controller structure (e.g., learning of the RB).

At the end of the corresponding section, some works are described that represent a hybridization of MOEAs, fuzzy logic, and neural networks.

C. Multiobjective Evolutionary Fuzzy Systems Designed for Fuzzy Association Rule Mining

The hybridization of MOEAs and fuzzy systems permits automatic knowledge extraction from data; therefore, data mining problems are one of the most important application domains for MOEFSs. Data mining has been treated as a synonym of knowledge discovery in databases (KDD) [31], [32], although it is a step of KDD. The high-level goals of data mining tend to be predictive, descriptive, or a combination of both.

A predictive approach focuses on accuracy in predictive ability and generates models that can be used to predict explicit values, based on patterns that are determined from known results. In prediction, a user may not care whether the model reflects reality as long as it has predictive power. One of the methods that are used in predictive models is supervised learning, which can create a function from training data, used to predict the output value for any valid input object. The predictive approach is applied in classification and regression, and in some cases, it can also be used in control problems.

On the other hand, the descriptive approach focuses on understanding the implicit data-generating process, searching for interesting patterns in existing data, without having any predefined target. The method that is used in this model is usually unsupervised learning, which differs from supervised learning in that there is no *a priori* output to train the model. This method is mainly applied to models that work with associative rules.

Finally, in some cases there are data mining applications demanding some degree of both predictive and descriptive approaches. A method which combines the mixed approach between descriptive and predictive is Subgroup Discovery [33].

One way to represent knowledge extracted with data mining techniques is by means of association rules [34], whose basic concept is to discover meaningful associations between different pairs of sets of attribute values. For example, the presence of a value of some set in a database element implies the presence of another value in another set. Since fuzzy systems can deal with imprecise knowledge, they can be successfully applied in the representation of this knowledge using fuzzy association rules [35].

In mining fuzzy association rules, the objectives are based on the quality of the extracted rules: these rules should be precise, general or specific enough, interesting, etc. Because of the large amount of metrics, MOEAs have been used successfully to mine fuzzy association rules.

The works included in this group mainly use a descriptive approach, i.e., description sets focused on making the data comprehensible and interpretable. Additionally, some works using the Subgroup Discovery approach will be described.

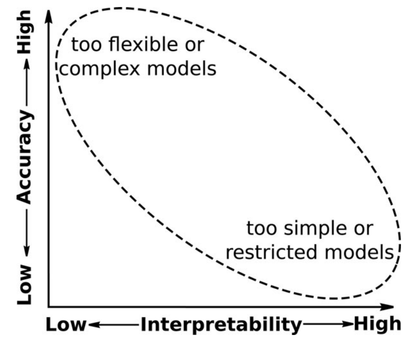

Fig. 2. Accuracy–interpretability tradeoff.

III. Multiobjective Evolutionary Fuzzy Systems Designed to Generate Fuzzy Rule-Based Systems With Different Accuracy–Interpretability Tradeoffs

The problem of improving accuracy while maintaining or even improving the interpretability of a fuzzy system is widely acknowledged in the community of MOEFSs; its presence was noted in the mid-1990s [19]. It is known that there is a point at which it is not possible to improve both the accuracy and interpretability of a fuzzy system at the same time. Therefore, in this framework an MOEA aims to find a set of feasible fuzzy systems with different tradeoffs between accuracy and interpretability (see Fig. 2).

Hereinafter, we describe contributions in which MOEFSs are designed to generate FRBSs with a good tradeoff between accuracy and interpretability, and we group them by following the second level of the taxonomy that is presented in Section II and explained in Section II-A.

A. Approaches to Performing Tuning

MOEAs can be used to perform the genetic tuning of FRBS components. Genetic tuning is applied as a postprocessing method, once the RB has been obtained, to refine the KB parameters [36]–[39] or to adapt the parameters of the inference engine [40]; therefore, the works belonging to this category have been divided into two subcategories: membership function tuning and inference parameter tuning. Moreover, in some cases the tuning process can be combined with a rule selection process, to improve the interpretability of the obtained model by removing unnecessary rules. This approach can be seen as a rule set tuning process, since it is applied to a previously defined RB.

1) Tuning of Membership Functions: An example of membership functions tuning process combined with a rule selection process can be found in [41], in which the authors present a postprocessing algorithm to improve the performance of linguistic FRBSs for regression problems. A specific MOEA is used to achieve a good balance between accuracy and complexity, improving accuracy by the tuning of membership functions, while reducing complexity by removing unnecessary rules. The proposed algorithm, which is called accuracy-oriented strength Pareto evolutionary algorithm 2 ($SPEA2_{ACC}$), is based on a particular modification of SPEA2 [42] and takes into account two objectives: accuracy, expressed by computing the mean squared error (MSE), and complexity, expressed as the number of selected rules. Rule selection and the tuning of membership functions are performed together, by coding both in the same chromosome. The $SPEA2_{ACC}$ concentrates the search on the Pareto zone that have the most accurate solutions with the least number of possible rules.

The same algorithm is extended in [43], in which six algorithms are considered to perform a rule selection from a given fuzzy rule set along with the tuning of the membership function parameters applied to regression problems. The nondominated sorting genetic algorithm II (NSGA-II) [44] and SPEA2 are used, along with two versions of NSGA-II proposed for general use, which concentrate the search on the Pareto knees. Two MOEAs for specific application to this concrete problem are applied. The first one is the $SPEA2_{ACC}$ proposed in [41], and the second one is its extension, i.e., $SPEA2_{ACC^2}$. All these algorithms improve two objectives: MSE and the number of rules.

In [45], a hybrid method for the identification of a Pareto-optimal fuzzy rule-based classifier (FRBC) is presented. The initial population is created in two steps: first a decision tree, which is generated through the classical C4.5 algorithm, is transformed into an FRBC. This way, relevant variables are selected, and an initial partition of the input space is performed. Afterward, the remaining population is created by randomly replacing some parameters of the initial FRBC. The tuning process is performed by applying the well-known NSGA-II, with polynomial mutation and simulated binary crossover (SBX) [46] as genetic operators. Three objectives are minimized: the number of misclassified patterns, the number of rules, and the total number of conditions in the rules. Each chromosome codifies an FRBC, including antecedents of the rules and parameters of the fuzzy sets.

An adaptation of the previous framework can be found in [47], in which FRBCs are used to model a bioareosol detector. As the metrics of accuracy, true positive (TP) and false positive (FP) rates were used instead of the commonly used misclassification rate, because of the uneven misclassification costs and class distributions of the collected data. Interpretability of the model is also a requirement, since it allows the bioareosol detector to be subsequently adjusted. Therefore, NSGA-II is applied to find FRBCs with a good tradeoff between objectives. The FP rate and the complement of the TP rate measure the accuracy, whereas transparency of fuzzy partitions is used for interpretability. The latter objective is expressed by the sum of three interpretability measures: the length of overlap and the length of discontinuity between fuzzy sets, proposed by Kim *et al.* [48], and the middle value penalty.

Another contribution to the tuning of DB parameters of FRBSs for regression problems can be found in [49]. In this work, the concept of context adaptation is used: context adaptation is a tuning process that exploits context-specific information to adapt a context-free model to a context-adapted FRBS. NSGA-II has been applied to the tuning of DB parameters, to maximize both the accuracy and interpretability of a linguistic FRBS. A novel index is, therefore, proposed, to provide a measure of interpretability, considering ordering, coverage, and distinguishability. The proposed index and the MSE are used as objectives of the EA.

The tuning of membership function parameters is tackled again in [29], in the framework of linguistic fuzzy models for regression problems. A novel relative index is proposed to help preserve the semantic interpretability of FRBSs while the tuning of membership functions is performed. The index, which is called GM3M, is the aggregation of three metrics that aim to maintain the original meanings of the membership functions as much as possible. In this paper, a tuning of membership function parameters is combined with a rule-selection mechanism, in order to also reduce the complexity of the fuzzy models. Therefore, an improved specific version of the well-known SPEA2, namely SPEA2-SI, including incest prevention and restarting, is proposed, and three objectives are considered: accuracy maximization, semantic interpretability maximization, and complexity reduction.

2) Tuning of Inference Parameters: Few works have taken into account the tuning of the inference engine [40]. In [50], a method is presented to concurrently learn the fuzzy inference operators and the RB of linguistic FRBSs, in order to obtain simpler, more compact yet still accurate linguistic fuzzy models. To this end, two MOEAs were used and adapted: SPEA2 and NSGA-II. The proposed MOEAs generate a set of FRBSs with different tradeoffs between interpretability and accuracy: The two objectives are expressed by the number of rules and the MSE, respectively.

In [51], an approach is proposed to tackle the interpretability-accuracy tradeoff in linguistic FRBSs with adaptive defuzzification. Adaptive defuzzification methods improve the accuracy of the system, but cause a loss of interpretability and increase complexity, due to the introduction of parameters in the defuzzification operator and weights associated with each rule. To quantify the interpretability of FRBSs with adaptive defuzzification, a novel index is proposed, which is the aggregation of two metrics: number of rules with weight and average number of rules triggered by each example. Afterward, an adaptation of NSGA-II is exploited in order to obtain a set of accurate and interpretable linguistic fuzzy models with adaptive defuzzification. Three objectives are minimized: the MSE, the number of final rules in the system, and the proposed interpretability index.

B. Approaches to Performing KB Learning

Besides the tuning of FRBS components, another possibility is to learn the KB or a part of it by means of MOEAs. We identify three approaches within this category: learning by rule selection, RB learning, and the simultaneous learning of KB components.

1) Approaches to Learning by Rule Selection: The first contributions to the application of MOEAs to linguistic FRBS generation with a good interpretability-accuracy tradeoff were proposed by Ishibuchi's group on multiobjective rule selection applied to learning. In their earlier works [19], [52], the authors use first-generation MOEAs (i.e., MOEAs without elitism) to perform a rule selection on an initial set of candidate rules as a two-stages learning process: candidate rule set generation and multiobjective rule selection. In the second stage, they consider two different objectives: maximization of the number of correctly classified training patterns and minimization of the number of selected rules; therefore, the obtained classification systems consist of a small number of linguistic rules. In [52], this rule selection method is extended to the case of classification problems with many continuous attributes, by using a prescreening procedure of candidate rules based on the number of antecedent conditions of each rule.

To better control the dimensionality problem, the authors add a third objective in [53]. An MOEA is used to extract a small number of fuzzy rules from numerical data, taking into account three objectives: to maximize the number of correctly classified training patterns, to minimize the number of fuzzy rules, and to minimize the total number of antecedent conditions. The MOEA presented in [19] is extended to a multiobjective genetic local search (MOGLS) algorithm, in which a local search procedure adjusts the selection process. Moreover, it is combined with a learning algorithm to obtain rule weights.

In [54], two multiobjective genetic-based approaches are applied, to obtain FRBCs with a good tradeoff between accuracy and complexity. The first approach was presented in [19], while the second one is a hybrid multiobjective genetics-based machine learning (GBML) algorithm, which is a hybridization between the Michigan [55], [56] and Pittsburgh [57] approaches. It considers the same three objectives as the previous model [53].

The same multiobjective GBML algorithm is used in [58], but in this contribution it is implemented taking advantage of the well-known NSGA-II and again consists of a hybrid version of the Michigan and Pittsburgh approaches: each fuzzy rule is represented by its antecedent fuzzy sets as an integer string of fixed length; then the concatenation of these strings represents an FRBC. The objectives remain the same as in [54].

In [59], NSGA-II is applied to the design of FRBCs belonging to the accuracy-complexity Pareto optimal front. The accuracy of each classifier is measured as the number of correctly classified training patterns, whereas the complexity is computed as the number of fuzzy rules and the total number of antecedent conditions. Finally, an ensemble classifier (also called a multiclassifier) is designed by combining nondominated FRBCs, and its performances are analyzed by performing computational experiments on six benchmark datasets that are taken from the University of California at Irvine (UCI) machine learning repository. The authors observe that the effect of combining several FRBCs is problem dependent and that an ensemble of classifiers with high diversity usually has better performances.

2) Approaches to Performing Rule Base Learning: Most of the approaches that are proposed to automatically learn the KB from numerical information focus on RB learning using a predefined DB.

In [60], an MOEA is used to generate FRBCs with a good tradeoff between the complexity of the rule systems and their reflection of the data. This MOEA uses a measure based on Area Under the receiver operating characteristic Curve (AUC) to determine how well the classifier reflects the data. Moreover, some concepts that are taken from SPEA2 are included: the fitness assignment of SPEA2 is used to avoid premature convergence, and an external archive is maintained to store the best individuals from all the solutions considered. In addition, a tailor-made representation scheme is used to preserve the comprehensibility of the rule systems, and a self-adaptation mechanism is included to reduce the number of free parameters. Three objectives are optimized: the accuracy, expressed as a measure based on the AUC, and complexity, computed as the number of rules and conditions.

An example of rule learning for regression problems is presented in [61], in which the authors propose a modified version of the well-known (2+2)Pareto archived evolution strategy (PAES), which is called (2+2)M-PAES, introduced in [62]. Unlike classical (2+2)PAES, which only uses mutation to generate new candidate solutions, (2+2)M-PAES exploits both crossover and mutation. This approach considers a predefined DB uniformly distributed and enables a large set of RBs to be derived, concurrently minimizing the accuracy and the complexity. The accuracy is computed as the root mean squared error (RMSE), whereas complexity is measured as the sum of the conditions which compose each of the antecedents of the rules included in the FRBS.

In [63], the accuracy–interpretability tradeoff is considered in the context of imbalanced classification problems. Usually, the accuracy of a classifier is measured as the percentage of correct classification, but this objective might not be suitable for problems that are characterized by highly imbalanced distributions of patterns. In this proposal, the authors applied the well-known NSGA-II to provide a set of binary FRBCs with a good tradeoff between complexity and accuracy. In this case, complexity is computed as the sum of the conditions in the antecedents of the classifier rules, whereas accuracy is expressed in terms of two objectives: sensitivity and specificity. These express how well the system classifies patterns belonging to the positive class and the negative class, respectively.

3) Approaches to Simultaneous Learning of KB Components: KB learning of linguistic FRBSs aims to learn the DB and RB concurrently. This approach tackles a very large search space, which is also difficult for EAs to handle. Some approaches have been proposed to learn concurrently the overall RB and DB.

In [64], the authors proposed a method for feature selection and DB learning, to obtain FRBCs composed of a compact set of comprehensible fuzzy rules with high classification ability. The DB learning involves both the number of labels for each variable (granularity) and the form of each fuzzy membership function. A nonlinear scaling function is used to adapt the fuzzy partition contexts for the corresponding granularity. This approach uses an MOEA to evolve the DB and considers a simple generation method to derive the RB. The MOEA has two goals: to improve the accuracy, by minimizing the classification error percentage over the training dataset, and to obtain a compact and interpretable KB, by penalizing fuzzy classifiers with large numbers of selected features and high granularity. The second objective is expressed by the product of the number of selected variables and their averaged granularity.

In [65], the authors proposed a technique to concurrently perform the RB identification and the DB learning of fuzzy models for regression problems. Two MOEAs are exploited to generate a set of linguistic FRBSs with different tradeoffs between accuracy and interpretability. The proposed approach can learn RBs and membership function parameters of the associated linguistic labels; therefore, the search space increases considerably. To manage the size of the search space, the linguistic two-tuple representation model [66] is included, which uses a reduced number of parameters to perform the symbolic translation of labels. The first MOEA is (2+2)M-PAES, and it is compared with the well-known NSGA-II. Two objectives are considered: the MSE and the number of antecedents activated in each rule.

The same (2+2)M-PAES is exploited in [67] to generate linguistic FRBSs for regression problems, with different tradeoffs between complexity and accuracy. The presented approach aims to learn the RB and the granularity of the uniform partitions defined by the input and output variables concurrently. Consequently, the concepts of virtual and concrete RBs are introduced: the former is defined by uniformly partitioning each linguistic variable with a fixed maximum number of fuzzy sets. The latter takes into account, for each variable, the number of fuzzy sets determined by the specific partition granularity of that variable. RBs and membership function parameters are defined by the virtual partitions and, whenever a fitness evaluation is required, they are mapped to the concrete partitions. Two objectives are considered: the accuracy of the FRBSs, measured as the MSE, and their complexity, computed as the number of propositions used in the antecedent of the rules contained in the concrete RB.

This work is extended in [68], in which the same MOEA is used to concurrently learn not only the RB and partition granularity but membership function parameters as well. The same approach is presented in [69], where a partition integrity index is proposed as a third objective. This index measures to what extent a partition is different from an initially interpretable one. Furthermore, in [28] a novel interpretability index is proposed, which combines RB complexity with DB integrity.

In [70], a specific MOEA, which is called Pitt-DNF, is proposed to obtain FRBSs for regression problems. The Pittsburgh approach is chosen; therefore, each chromosome encodes a complete set of fuzzy rules. Antecedents of rules are represented in disjunctive normal form (DNF), i.e., each input variable can take an OR-ded combination of several linguistic terms as a value, and the different input linguistic variables are combined by an AND operator. Nevertheless, the authors wrongly call conjunctive normal form these kinds of fuzzy rules. This representation provides a high degree of compactness and improves the interpretability of fuzzy models, but the combination of the Pittsburgh approach with DNF-type fuzzy rules causes some problems to generate the rules themselves. The proposed learning algorithm, which is based on NSGA-II, has been developed to avoid the generation of DNF-type fuzzy rule sets with these problems, and it gives a set of solutions with different tradeoffs between complexity, computed as the number of DNF rules, and accuracy, measured by the MSE. One crossover operator and two mutation operators were specifically designed to take into account the particular representation of fuzzy rules, thus avoiding inconsistency, redundancy, overgenerality, and incompleteness in fuzzy rules.

In [71], an MOEA is proposed to learn the granularities of fuzzy partitions, tune the membership function parameters, and learn the fuzzy rules of a linguistic FRBS for regression problems. A two-step evolutionary approach is applied: the fuzzy models are initialized using a method that combines the benefits of an ad hoc RB generation algorithm and decision-tree algorithms, with the aim to reduce the search space. The initial population is then optimized by an MOEA that reduces the number of rules, rule conditions, membership functions, and input variables. The MOEA is based on NSGA-II, and the original genetic operators are replaced with new ones that take into account dynamic constraints to ensure the transparency of fuzzy partitions. Two objectives are optimized: accuracy, expressed as the MSE, and complexity, computed as the total rule length (number of active rule conditions).

In [72], a two-stage approach to obtain linguistic KBs in classification problems is proposed, based on the multiobjective fuzzy rule selection presented in [53] and by including a lateral tuning [39] within the same process and by considering the same three objectives: to maximize the number of correctly classified training patterns, to minimize the number of fuzzy rules, and to minimize the total number of antecedent conditions. The first stage determines appropriate granularities for the DB and a set of candidate rules. The second stage performs multiobjective rule selection and tuning, based on using NSGA-II to obtain the final RB and the appropriate DB parameters.

A recent proposal can be found in [73], where the authors focus on the scalability issue of linguistic FRBSs in 17 regression problems. The first stage uses an improved MOEA (based on SPEA2) to perform an embedded genetic DB learning including feature selection, granularities, and the reduced lateral displacement of fuzzy partitions in order to control the dataset dimensionality and obtain a reduced KB. For each DB definition an ad hoc RB is derived by adding a cropping mechanism to avoid large RBs and to reduce the required computation time. Two minimization objectives are used: MSE and number of rules. Finally, a postprocessing stage for fine tuning and rule selection is applied to the obtained KBs using the same objectives. A speeded-up version of a previous MOEA, namely exploration–exploitation-based SPEA2 ($SPEA2_{E/E}$), is presented by including a new approach to fast fitness estimation which only uses a small percentage of the training data. Since this mechanism is proposed for any kind of EA, the authors also include it in the first stage in order to address the problem of large datasets (many-instance datasets).

In [74], Alonso *et al.* propose embedding the high interpretable linguistic knowledge (HILK) heuristic method [75] in a three-objective EA, with the aim to get a good accuracy–interpretability tradeoff when building FRBCs. The well-known NSGA-II algorithm is employed, using two-point crossover and Thrift's mutation [76]. Three criteria are optimized: accuracy, by maximizing the right classification rate; readability, by minimizing the total rule length; and comprehensibility, by minimizing the average number of rules fired at the same time (average fired rules—AVR). Each chromosome includes a number of genes equal to the number of input variables, and each gene represents the number of linguistic terms defined for the related input. Recently, this proposal has been extended in [77] by considering a novel comprehensibility index called the logical view index (LVI), which estimates how much an RB satisfies logical properties. In this novel version, the AVR is substituted by the LVI as a better FRBCs comprehensibility measure. Finally, in [78], both LVI and AVR indices are considered. The proposed evolutionary framework is used to set up two independent experimental sessions with two objectives: classification rate versus AFR and classification rate versus LVI. The study aims to find possible relationships between AFR and LVI, showing that the AFR minimization implies the LVI minimization, while the opposite is not verified.

C. Approaches That Deal With TSK Fuzzy Rule-Based Systems

TSK fuzzy models provide a powerful tool to model complex nonlinear systems, as multiple submodels (typically linear models) are combined to describe the global behavior of the system. The resulting model is often more difficult to interpret, and few works can be found on this topic.

In [79], a technique based on a hierarchical MOEA [80], which is derived from MOGA [81], is proposed to construct TSK fuzzy models [11] from data, considering both their accuracy and interpretability. The initial model is generated through a two-step procedure: A fuzzy clustering method is used to preprocess the training data and to construct the rule antecedents, and then the recursive least-squares (RLS) method is applied to determine the consequent rule. Finally, the hierarchical MOEA is exploited to obtain the optimized fuzzy models, for regression problems. A hierarchical chromosome formulation is used so that it can perform the simultaneous optimization of rule antecedents and number of rules, whereas consequents are obtained with the RLS method. A two-level hierarchical structure is used: control genes and parameter genes. Considering that there are two types of genes in the chromosome, a multipoint crossover is applied for control genes, whereas for the parameter genes which are represented in real numbers, BLX-α crossover is applied. During the optimization, an interpretability-driven RB simplification is applied to reduce the search space. Five objectives are optimized: the MSE for accuracy, the total number of fuzzy sets and the number of fuzzy rules for compactness, a purposely defined aggregate index for both completeness and distinguishability, and, finally, an appropriate equation for nonredundancy.

In [82], a novel coevolutionary algorithm [83] is proposed to improve the performance of TSK fuzzy systems in regression problems. This algorithm is called the Pareto multiobjective cooperative coevolutionary algorithm (PMOCCA). The fuzzy system is decomposed into two species: antecedents of fuzzy rules and parameters of fuzzy sets. To obtain a good initial fuzzy system, a modified fuzzy clustering algorithm is used. Afterward, the PMOCCA and some interpretability-driven simplification techniques are used to evolve the initial fuzzy system with three objectives: accuracy of the system, the number of fuzzy rules, and the number of antecedents in each fuzzy rule.

The problem of the tradeoff between accuracy and complexity in TSK fuzzy systems is also faced in [84], in which a specific version of NSGA-II is proposed to determine a Pareto-optimal set of fuzzy models for regression problems. In particular, two competing objectives are addressed: the accuracy, measured by the normalized RMSE, and the complexity, expressed by the number of fuzzy rules. The specialization of the algorithm is obtained first by using several heuristics to obtain a good initialization of the population, and second by designing crossover and mutation operators specific to the problem.

In [85], a multiobjective neuroevolutionary algorithm (MONEA) is proposed to obtain a parameter estimation of TSK fuzzy models for regression problems. Neural network-based techniques and ad hoc techniques for interpretability improvement are included in the MOEA to increase the efficacy of the algorithm: the fuzzy model is defined by a radial basis function neural network [86], [87]. The number of neurons in the hidden layer of the neural network is equal to the number of rules in the fuzzy model, and the firing strength of the ith neuron in the hidden layer matches the firing strength of the ith rule. The neurons in the output layer perform the computation for the function that is described in the consequents of the fuzzy model. The MONEA considers four objectives: accuracy, computed as the MSE, transparency, for which the similarity among distinct fuzzy sets is considered, and compactness, expressed by the number of rules and the number of antecedents in the fuzzy model.

Another proposal can be found in [88], in which the authors used a hybrid technique to optimize the structure of TSK fuzzy systems for regression problems. First, a backpropagation algorithm is applied to optimize the membership function parameters and the parameters of fuzzy rules. In a second phase, NSGA-II is used to perform a fine tuning of parameters and to select the optimal number of fuzzy rules. The algorithm considers two objectives: the system's accuracy, computed as the MSE, and complexity, defined by the number of active fuzzy rules in the RB.

In [89], a regression problem named the ocean color inverse problem is approached by using the (2+2)M-PAES to optimize TSK FRBSs. The evolutionary optimization roughly identifies the structure of the fuzzy models, and then a tuning process is performed: TSK FRBSs are implemented as an artificial neural network, and by training the neural network, the parameters of the fuzzy model are adjusted. The result is a set of fuzzy models with different tradeoffs between accuracy and complexity.

A recent contribution has been presented in [90], in which the authors first analyze the time complexity for both the generation and the evaluation of TSK FRBSs. Since the identification of the rule consequent parameters is one of the most time-consuming phases in TSK FRBS generation, a simple and effective technique is proposed for speeding it up. Then, this approach is included in the optimization process of the structure of TSK fuzzy systems for regression problems. (2+2)M-PAES is applied, and one-point crossover and three appropriately defined mutation operators are used. Two objectives are optimized: the MSE as a measure of accuracy and the total number of conditions different from *don't care* as a measure of complexity.

D. Summary of Multiobjective Evolutionary Fuzzy Systems Designed to Generate Fuzzy Rule-Based Systems With Different Accuracy–Interpretability Tradeoffs

In order to give an overview of the contributions that are described so far, Table I presents a summary of the works that deal with the accuracy–interpretability tradeoff of FRBSs. Papers are grouped considering the components of the KB that are optimized and within each group they appear in chronological order. For each paper, the type of FRBS approach is shown, together with the kinds of rules. The number and type of the objectives are reported together with the name of the MOEA, its generation type, and the kind of proposal (novel, general use or based on a previous MOEA). The repetition of the objective type means the presence of two different optimized measures for the same objective. In the last column, the type of application problem is briefly described.

Except for some earlier works, the greater part of the approaches use a second-generation MOEA (i.e., MOEA with elitism) to tackle the accuracy–interpretability tradeoff: in fact, the introduction of the concept of elitism is essential for the convergence of the algorithms. Moreover, the concept of interpretability becomes more complex and complete over the years: earlier contributions considered interpretability only in terms of complexity, whereas more recently, semantic interpretability has been studied in depth and included in the optimization process.

Looking at the FRBS approach, it is evident that Mamdani FRBSs are used more than TSK ones, probably due to the intrinsic interpretability of the Mamdani model. Finally, we can remark that earlier contributions scarcely considered the problem of learning the whole KB, which is progressively considered more often in the latter contributions.

IV. Multiobjective Evolutionary Fuzzy Systems Designed For Multiobjective Control Problems

FLCs are one of the most common applications of fuzzy logic. An FLC includes a set of linguistic control rules that are related by the dual concepts of fuzzy implication and the compositional rule of inference [91]. No fixed process to design a fuzzy controller exists, and the appropriate fuzzy parameters have to be chosen on the basis of an experimental study of the control objective. To overcome this difficulty, the application of EAs was proposed for the design of FLCs [92]–[94]. Two problems arise during this process: The first issue concerns how to establish the structure of the controller; second, the numerical values of the controller's parameters have to be chosen.

Many contributions can be found in the literature on the use of EAs to obtain the optimal design of FLCs, both for tuning and learning tasks. Most of them take into account only one performance objective. The first multiobjective approaches were carried out by combining several performance objectives into a single one, by using a weighting approach. Afterward, more objectives were included in the optimization process with the aim of considering not only different performance measures, but also characteristics such as time constraints, robustness and stability requirements, comprehensibility, and the compactness of the obtained controller. EAs have been used either for offline or for online design of FLCs, although in the latter case the computation time is sometimes a critical issue. Further information on EAs applied to FLCs can be found in [95].

In the following, we will analyze the existing works on the application of MOEFSs to fuzzy control, considering both categories presented in Section II-B. They are controller parameters' identification and the learning of controller structure. Unless otherwise specified, the contributions use a Mamdani-type FLC.

A. Controller Parameters' Identification

The first approach aims to modify the parameters that affect the controller's performance once an initial design of the FLC is established. Tuned parameters can be the scaling factors for each variable, the shape of fuzzy sets representing the meaning of linguistic values and the selected IF–THEN rules. This approach permits the reduction of the computational load required, since the search space is smaller than the one considered when learning all the components together. Nevertheless, since the parameters and structure of fuzzy models are strictly related, the obtained solutions are affected by the initial system definition.

One of the first works on the use of the first-generation MOEA for the optimization of an FLC is presented in [96]. A Mamdani-type fuzzy system is designed for the vibration control of a civil engineering structure in seismic zones. Consideration of the building performance includes both the safety and the comfort level of the user. The former issue is achieved by minimizing the peak displacement, while the latter one is achieved by minimizing the peak acceleration. The tradeoff between the two objectives is handled using a two-branch tournament GA that provides a set of Pareto optimal solutions and optimizes the parameters of the input and output membership functions. Each membership function is represented by a generalized bell-shaped function that is defined by three values. One-point crossover is employed, and the mutation is performed on a bit-by-bit basis, with a certain probability.

A similar approach is undertaken in [97], where a hybrid control system (using active and passive control strategies) is proposed for the structural vibration control of buildings. A tuned mass damper and an active mass driver are used as respective the passive and active control components of the hybrid control system. To control the active mass driver, an FLC is used, and the two-branch tournament GA is applied to the optimization of the parameters of the input and output membership functions. In [98], a further objective is added. A three-branch tournament GA is used this time, in which the minimization of peak displacement, acceleration, and rotation of the structure about its vertical axis are considered as the three objective functions.

In [99], the same approach is used for the optimization of an FLC that drives an active tuned mass damper toward the response control of wind-excited tall buildings. Furthermore, in [100], the authors improve the proposal presented in [97] by

TABLE I
SUMMARY OF THE PROPOSALS ON MOEFSS DESIGNED TO GENERATE FRBSS WITH GOOD ACCURACY–INTERPRETABILITY TRADEOFFS

		A-I trade-off			FRBS approach		Objectives		MOEA			
		Authors	Ref.	Year	Rules	Type	#Obj.	Type	Name	Gen.	Type	Problem type
DB tuning (including rule set learning)	Membership functions tuning	Wang et al.	[79]	2005	TSK	Ling. *	5	A+C+C+S+S	MOHGA	1st	I ∘	Reg.
		Alcalá et al.	[41]	2007	Mam.	Ling.	2	A+C	$SPEA2_{ACC}$	2nd	I •	Reg.
		Gonzalez et al.	[84]	2007	TSK	Scat.	2	A+C	NoN.	2nd	I †	Reg.
		Gomez et al.	[85]	2007	TSK	Scat.	4	A+C+C+S	MONEA	2nd	N	Reg.
		Pulkkinen et al.	[45]	2008	Mam.	Ling.	3	A+C+C	NSGA-II	2nd	G	Clas.
		Pulkkinen et al.	[47]	2008	Mam.	Ling.	3	A+A+S	NSGA-II	2nd	G	Clas.
		Guenounou et al.	[88]	2009	TSK	Ling. *	2	A+C	NSGA-II	2nd	G	Reg.
		Gacto et al.	[43]	2009	Mam.	Ling.	2	A+C	various	2nd	G	Reg.
		Botta et al.	[49]	2009	Mam.	Ling.	2	A+S	NSGA-II	2nd	G	Reg.
		Gacto et al.	[29]	2010	Mam.	Ling.	3	A+C+S	SPEA2-SI	2nd	I •	Reg.
	I.P. tuning	Marquez et al.	[50]	2009	Mam.	Ling.	2	A+C	various	2nd	I †•	Reg.
		Marquez et al.	[51]	2010	Mam.	Ling.	3	A+C+S	NoN.	2nd	I †	Reg.
KB learning	Learn. by rule sel.	Ishibuchi et al.	[19], [52]	1997,1998	Mam.	Ling.	2	A+C	NoN.	1st	N	Clas.
		Ishibuchi et al.	[54]	2001	Mam.	Ling.	3	A+C+C	GBML	1st	N	Clas.
		Ishibuchi et al.	[53]	2004	Mam.	Ling.	3	A+C+C	MOGLS	1st	N	Clas.
		Alcalá et al.	[72]	2011	Mam.	Ling.	3	A+C+C	NSGA-II	2nd	G	Clas.
		Ishibuchi et al.	[59]	2006	Mam.	Ling.	2	A+C	NSGA-II	2nd	G	Clas.
		Ishibuchi et al.	[58]	2007	Mam.	Ling.	3	A+C+C	GBML	2nd	I †	Clas.
	RB learning	Setzkorn et al.	[60]	2005	Mam.	Ling.	3	A+C+C	NoN.	2nd	I •	Clas.
		Cococcioni et al.	[61]	2007	Mam.	Ling.	2	A+C	(2+2)M-PAES	2nd	I ⋆	Reg.
		Xing et al.	[82]	2007	TSK	Ling. *	2	A+C	PMOCCA	2nd	N	Reg.,Ts.
		Ducange et al.	[63]	2010	Mam.	Ling.	3	A+A+C	NSGA-II	2nd	G	Imb. Clas.
	Simultaneous learning of KB components	Cordón et al.	[64]	2003	Mam.	Ling.	2	A+C	NoN.	1st	N	Clas.
		Cococcioni et al.	[89]	2008	TSK	Scat.	2	A+C	(2+2)M-PAES	2nd	I ⋆	Reg.
		Alcalá et al.	[65]	2009	Mam.	Ling.	2	A+C	(2+2)M-PAES	2nd	I ⋆	Reg.
		Antonelli et al.	[67]	2009	Mam.	Ling.	2	A+C	(2+2)M-PAES	2nd	I ⋆	Reg.
		Antonelli et al.	[68]	2009	Mam.	Ling.	2	A+C	(2+2)M-PAES	2nd	I ⋆	Reg.
		Casillas et al.	[70]	2009	DNF-rules	Ling.	2	A+C	NoN.	2nd	I †	Reg.
		Pulkkinen et al.	[71]	2010	Mam.	Ling.	2	A+C	NoN.	2nd	I †	Reg.
		Alonso et al.	[74]	2010	Mam.	Ling.	3	A+C+S	NSGA-II	2nd	g	Clas.
		Cannone et al.	[77]	2011	Mam.	Ling.	3	A+C+S	NSGA-II	2nd	g	Clas.
		Cannone et al.	[78]	2011	Mam.	Ling.	2	A+S and A+S	NSGA-II	2nd	g	Clas.
		Cococcioni et al.	[90]	2011	TSK.	Scat.	2	A+C	(2+2)M-PAES	2nd	I ⋆	Reg.
		Antonelli et al.	[69]	2011	Mam.	Ling.	3	A+C+S	(2+2)M-PAES	2nd	I ⋆	Reg.
		Antonelli et al.	[28]	2011	Mam.	Ling.	2	A+(C+S)	(2+2)M-PAES	2nd	I ⋆	Reg.
		Alcalá et al.	[73]	2011	Mam.	Ling.	2	A+C	NoN.	2nd	I †	Reg.

I.P.=inference parameters, Mam.=Mamdani, TSK=Takagi–Sugeno–Kang, Ling.=linguistic, Scat.=scatter, ∗in the antecedent;
A=accuracy, C=complexity, S=semantic aspects;
NoN.=no name, N=novel algorithm, I=improved version, G=general use;
Clas.=classification, Reg.=regression, Ts.=time series, Imb.=imbalanced;
†NSGA-II based, ⋆ PAES based, ∘ MOGA based, • SPEA2 based.

adding an active control system to the hybrid control system. The overall system is driven by an FLC, whose parameters are optimized by means of the two-branch tournament GA, presented in the previous works.

Further works use the first-generation MOEA to tune the parameters of the membership functions of an FLC. In [101], a hierarchical MOGA-based approach is used to tune fuzzy scheduling controllers for a gas turbine engine. The engine should satisfy nine large-signal performance criteria (e.g., steady-state accuracy, transient accuracy, disturbance rejection, stability, stall margin, structural integrity, engine degradation, etc.). Once an initial suitable fuzzy scheduling controller is designed, parameters of membership functions and scaling factors are tuned to meet the former criteria.

In [102], an MOGA-based approach is presented to tune an FLC for a solid oxide fuel cell power plant. The obtained model achieves fast transient responses and has very low total harmonic distortion in output current steady-state operation. To improve the fuzzy structure of the controller, a tuning process adapts the parameters of membership functions and scaling factors. Fuzzy sets are defined by the center points of normalized, triangular membership functions. Objectives are described by a system of equations that represent the harmonics to be minimized.

With regard to the use of second-generation MOEAs, in [103], the authors investigate the use of smart base-isolation strategies to reduce structural damage that is caused by severe loads. A friction pendulum system and a magnetorheological damper are employed in a smart base-isolation system, and an FLC is used to modulate the magnetorheological damper. The classic NSGA-II is used to optimize parameters of membership functions and to find appropriate fuzzy rules for the FLC. Gaussian membership functions are used for all input and output variables of the FLC. The shapes of Gaussian membership functions are defined by two parameters and are coded into the chromosome with a real-valued representation. The optimization process aims to minimize root mean squared structural acceleration and base drift.

This problem is tackled again in [104], in which a novel control technique is proposed, by utilizing a hierarchical structure of FLCs. The structure consists of two lower level controllers and a higher level supervisory controller. Lower layer controllers are optimized by NSGA-II, considering four objectives: reduction of peak superstructure acceleration, peak isolation system deformation, RMSE of superstructure acceleration, and RMSE of isolation system deformation. Gaussian membership functions are used for all input and output variables of the FLC, as in the previous contribution.

In [105], an FLC is designed to manage two magnetorheological dampers for the mitigation of seismic loads. NSGA-II with controlled elitism is used for the optimization of FLC parameters. Fuzzy sets of input and output variables are represented by Gaussian membership functions, which are described by two parameters. These parameters are coded in the chromosome by means of floating point values. The overall optimization process aims to maximize four objective functions: peak interstory drift, peak acceleration, RMSE of interstory drift, and RMSE of acceleration.

In [106], the authors present a multiobjective evolutionary process to tune the fuzzy membership functions of a fuzzy visual system for autonomous robots. This fuzzy visual system is based on a hierarchical structure that includes three different linguistic FRBCs. The combined action of these classifiers allows robots to detect the presence of doors in the images that are captured by their cameras. The DB of the whole fuzzy visual system is coded in a single chromosome, which comprises the four parameters defining each trapezoidal-shaped membership function. Blend crossover (BLX-α) [107] and random mutation are considered as genetic operators, whereas the conflicting objectives to be optimized are the TP and FP detection rates. Different single (a generational GA and CHC) and multiobjective (SPEA, SPEA2, NSGA-II) evolutionary algorithms are considered and compared, with NSGA-II obtaining the best performance.

In [108], the parameters of an adaptable hierarchical TSK fuzzy controller for blinds are optimized by NSGA-II, considering two objectives: energy consumption and thermal comfort. The fuzzy sets are represented by triangular membership functions whose parameters are optimized. The performances of the FLC are tested by means of software for dynamic simulation of indoor climate and energy.

In [109], an MOEA based on SPEA2 is developed to optimize the parameter of an FLC that aims to improve the water quality of a sewage treatment plant. The FLC uses ten parameters for its operation, and each chromosome codifies a set of parameters. Depending on these parameters, the controller decides when to activate a blower in the aeration tank, in order to keep the water clean. Water quality is based on different criteria; therefore, the optimization process tries to minimize the concentrations of three chemical compounds.

In [93], the authors proposed a tuning process combined with a rule selection process, to improve the performance of FLCs for the control of heating, ventilating, and air conditioning (HVAC) systems, including several performance criteria such as energy performance, stability, and indoor comfort requirements. The technique is based on $SPEA2_{E/E}$ and aims to obtain a more accurate controller by forcing the removal of unnecessary rules and biasing the search through those solutions that satisfy the performance objective to a higher degree. Two objectives are considered: maximizing the performance, which is expressed by aggregating five quality measures, and minimizing the complexity, which is computed as the number of rules obtained.

B. Learning of Controller Structure

Learning of controller structures is used for the generation of an FLC in situations where a reasonable set of rules is not immediately apparent. These kinds of approaches are able to take into account the synergy between the RB and DB, but they involve a heavier computational burden due to the increase in the search space.

One of the first works in this branch is [110]. An FLC for a nonlinear missile autopilot is designed using NSGA. Both the membership functions' distribution and the RB of the FLC are determined. The design process minimizes four objectives: the steady-state error, the overshoot, the settling time, and the rising time.

In [111], a specific MOEA is presented for the online design of the structure of a fuzzy speed controller for a dc motor motion control platform. The optimization involves three objectives to be minimized: the current tracking error, the velocity tracking error, and the power consumption of the system.

A medical application is presented in [112], where an MOEA is used to design FLCs to adjust the amount of drug dosage necessary to achieve the required neuromuscular blockade level in patients during surgery. The evolutionary approach is based on SPEA2 and considers two goals: the optimization of the amount of the drug required and the minimization of the complexity of the obtained FLC so that the undertaken control decision can be explained in natural language.

Beyond the works presented previously, there are some contributions that use a hybrid approach of fuzzy systems, neural networks and GAs, in order to automatically construct a controller. For example, in [113] an intelligent combustion controller is designed to handle an incineration process, by integrating different soft computing approaches. The proposed methodology applies three techniques simultaneously: A representative state function is modeled using a GA and a neural network. Then, this model is used as surrogate of the plant, and a specific first-generation MOEA is applied to obtain a set of FLCs, represented by TSK-type control rules. Finally, the control RB is improved by a tuning process. In this specific application, two goals are considered: effluent quality and heat recovery.

In [114], a gain-scheduling adaptive control scheme for nonlinear plants is presented. The controller is based on fuzzy systems, neural networks, and GAs. A fuzzy proportional integral (PI) controller is optimally designed using a specific MOEA to satisfy three objectives: minimizing overshoot time, minimizing settling time, and smoothing output response. Then, the backpropagation algorithm is applied to design a neural gain

TABLE II
SUMMARY OF THE PROPOSALS ON MOEFSS FOR MULTIOBJECTIVE FUZZY CONTROL PROBLEMS

	Fuzzy Control			FRBS approach			MOEA			
	Authors	Ref.	Year	Rules	Type	#Obj.	Name	Gen.	Type	Application Framework
CONTROLLER PARAMETERS' IDENTIFICATION	Ahlawat et al.	[96]	2001	MAM.	LING.	2	NoN.	1st	I ⋆	BUILDING VIBRATION
	Ahlawat et al.	[97], [99]	2002,2004	MAM.	LING.	2	NoN.	1st	I ⋆	BUILDING VIBRATION
	Ahlawat et al.	[98]	2002	MAM.	LING.	3	NoN.	1st	I ⋆	BUILDING VIBRATION
	Chipperfield et al.	[101]	2002	MAM.	LING.	9	NoN.	1st	N	GAS TURBINE ENGINE
	Ahlawat et al.	[100]	2004	MAM.	LING.	2	NoN.	1st	I ⋆	BUILDING VIBRATION
	Jurado et al.	[102]	2005	MAM.	LING.	16	NoN.	1st	I ∘	SOLID OXIDE FUEL CELL
	Kim et al.	[103]	2006	MAM.	SCAT.	2	NSGA-II	2nd	G	BASE-ISOLATION SYSTEM
	Kim et al.	[104]	2007	MAM.	SCAT.	4	NSGA-II	2nd	G	BASE-ISOLATION SYSTEM
	Shook et al.	[105]	2008	MAM.	LING.	4	NSGA-II CE	2nd	I †	SEISMIC LOADS MITIGATION
	Muñoz et al.	[106]	2008	MAM.	LING.	2	VARIOUS	2nd	G	FUZZY VISUAL SYSTEM FOR ROBOTS
	Daum et al.	[108]	2010	TSK	SCAT.	2	NSGA-II	2nd	G	HVAC SYSTEMS
	Ebner et al.	[109]	2010	‡	‡	3	NoN.	2nd	I •	WATER TREATMENT
	Gacto et al.	[93]	2012	MAM.	LING.	2	$SPEA2_{E/E}$	2nd	I •	HVAC SYSTEMS
LEARNING FLC STRUCT.	Blumel et al.	[110]	2001	MAM.	LING.	4	NSGA	1st	N	MISSILE AUTOPILOT
	Chen et al.	[113]	2002	TSK	LING. ∗	2	NoN.	1st	N	INCINERATION PROCESS
	Stewart et al.	[111]	2004	MAM.	LING.	3	NoN.	1st	N	DC MOTOR MOTION CTRL.
	Serra et al.	[114]	2006	MAM.	LING.	3	NoN.	2nd	N	NONLINEAR PLANTS
	Fazendeiro et al.	[112]	2007	MAM.	LING.	2	NoN.	2nd	I •	DRUG DOSAGE FOR SURGERIES

MAM.=Mamdani, TSK=Takagi–Sugeno–Kang, LING.=linguistic, SCAT.=scatter, ∗in the antecedent, ‡Patented FLC, not available information;
A=accuracy, C=complexity, S=semantic aspects;
NoN.=no name, N=novel algorithm, I=improved version, G=general use;
⋆ 2-branch tournament GA, ∘ MOGA based, †NSGA-II based, • SPEA2 based.

scheduler with the aim to tune the optimal parameters of the fuzzy PI controller at some operating points.

C. Summary of Multiobjective Evolutionary Fuzzy Systems Designed for Multiobjective Control Problems

All contributions on MOEFSs that are designed for fuzzy control are grouped in Table II. Papers are divided based on the aspects of the controller that are considered by the optimization process. A description of this type of table is given for Table I. In almost all cases, the objectives express a performance measure; therefore, the objective type does not appear in this table. Because of the various application fields of FLCs, the last column contains a brief description of the application framework. Within each group, papers are sorted in chronological order.

In most cases, the proposal deals with the postprocessing of FLC parameters, since it is the simplest approach and requires a reduced search space. Earlier works consider first-generation algorithms, and only very recently the best known second-generation MOEAs have been applied. Finally, in almost all papers, a Mamdani-type FRBS is used.

V. MULTIOBJECTIVE EVOLUTIONARY FUZZY SYSTEMS DESIGNED FOR FUZZY ASSOCIATION RULE MINING

The knowledge which is extracted by the mining process can be represented in several ways, for example, using association rules. A general association rule is defined as an implication $X \Rightarrow Y$, where both X and Y are defined as sets of attributes. This implication is interpreted as follows: "for a specified fraction of the existing transactions, a particular value of attribute set X determines the value of attribute set Y as another particular value under a certain confidence," where a transaction consists of a set of items I.

Two classic concepts are involved in association rules: support, which is the percentage of transactions that contains both X and Y, and confidence, that is, the ratio between the support of $X \cup Y$ and the support of X. Thus, the problem of association rule mining [34] consists of finding all association rules that satisfy user-specified minimum support and confidence. Early works used Boolean association rules, which consider only whether an item is present in a transaction or not, without evaluating its quantity. To take into account this aspect, fuzzy association rules [35] were introduced.

In the following, we describe those contributions that apply MOEFSs to fuzzy association rule mining. Then, a brief summary of the existing works is provided.

A. Description of the Existent Contributions

Fuzzy association rule extraction can be performed using MOEAs, as they obtain good results when dealing with problems involving several measures that could be contradictory to some degree. Moreover, they could also include interpretability concepts, since fuzzy association rules can explain the associations they represent.

For example, in [115], a specific Pareto-based multiobjective evolutionary approach is presented for mining optimized fuzzy association rules. Two different coding schemes are proposed: the first one tries to determine the appropriate fuzzy sets in a prespecified rule, also called certain rule. In such cases, each individual represents the base values of membership functions of a quantitative attribute in the DB. The second coding scheme tries to find both rules and their appropriate fuzzy sets. In both approaches, three objectives are maximized: support, confidence, and comprehensibility of fuzzy association rules, where the last one is expressed by a measure related to the number of attributes in a rule.

TABLE III
SUMMARY OF THE PROPOSALS ON MOEFSS FOR MINING FUZZY ASSOCIATION RULES

Fuzzy association rule mining			Objectives		MOEA		
Authors	Ref.	Year	#Obj.	Description	Name	Gen.	Type
Kaya et al.	[115]	2006	3	↑Sup. + ↑Con. + ↓Att.	NoN.	2nd	N
Alhajj et al.	[116]	2008	2	↑LI + ↓Tim.	NoN.	2nd	I •
Chen et al.	[117]	2008	2	↑L1I + ↑Sui.	NoN.	1st	I ◦
Thilgam et al.	[118]	2008	2	↑Sup. + ↑Con.	MOGA	1st	G
Casillas et al.	[119]	2009	3	↓Err. + ↓DNF-FR + ↓MAM-FR	NoN.	2nd	I †
Carmona et al.	[120] ∗	2010	3	↑Sup. + ↑FCon. + ↑Unu.	NMEEF-SD	2nd	I †

∗Applied for Subgroup Discovery;
Con.=confidence, Sup.=support, Tim.=time, Err.=error, LI=#large itemsets, L1I=#large 1-itemsets, Att.=#attributes, Sui.=suitability, DNF-FR=#DNF-type fuzzy Rules, MAM-FR=#equivalent Mamdani-type fuzzy rules, Unu.=unusualness, FCon.=fuzzy confidence,
↑ maximize, ↓ minimize, NoN.=no Name, N=novel algorithm, I=improved version, G=general use; †NSGA-II based, ⋆ PAES based,
◦ MOGA based, • SPEA based.

A fuzzy data mining approach is presented in [116] for the single-minimum-support fuzzy-mining problem. An MOGA-based algorithm is proposed to extract both membership functions and association rules from quantitative transactions. The algorithm tries to maximize two objectives. The first is the suitability of membership functions, through a combination of coverage and overlap factors. This measure is used to reduce the membership functions that are redundant or too separate. The second objective is the total number of large 1-itemsets in a given set of minimum support values. Since a larger number of 1-itemsets will usually result in a larger number of all of the itemsets with a higher probability, this implies more interesting association rules. Thus, this metric expresses the interestingness of a rule.

The earlier proposals in fuzzy association rule mining assumed that the number of fuzzy sets is pre-specified. In [117], an automated clustering method is proposed, which aims to automatically cluster values of a quantitative attribute, in order to obtain a large number of large itemsets in less time. The method uses an MOEA which is based on SPEA, and the optimization process considers two objectives. The first is to maximize the number of large itemsets with respect to a given minimum support value, since a large itemset potentially leads to the discovery of some interesting fuzzy association rules. The second objective is to minimize the time required to find all large itemsets in a given database.

In [118], a technique to mine optimized fuzzy association rules is proposed, to detect intrusions in a network. The proposed framework aims to concurrently identify fuzzy attributes and to define the membership functions by exploiting clustering techniques. Afterward, MOGA [81] is applied to generate and optimize fuzzy association rules of different orders. The optimization process tries to maximize two objectives: confidence, which represents the strength of a rule, and support, which in this case identifies the generality of a rule.

A particular approach which is focused on predictive induction is presented in [119], in which an MOEA is used to derive fuzzy association rules from uncertain data for consumer behavior modeling. Rules are codified with DNF-type fuzzy rules. The proposed framework considers data collection, data mining, and, finally, knowledge interpretation. During the mining process, an evolutionary scheme which is based on NSGA-II is applied, and three objectives are minimized. The accuracy is expressed by the approximation error; the complexity is represented by the number of DNF-type fuzzy rules. This second objective does not completely assess the interpretability of the fuzzy system, since the internal structure of each DNF-type fuzzy rule is not considered. Thus, a third objective is added that measures the number of equivalent Mamdani-type fuzzy rules for each DNF-type fuzzy rule.

Beyond predictive and descriptive induction, there are mixed techniques that combine the characteristics of both types of induction. An example is Subgroup Discovery [33], which aims to extract descriptive knowledge from data that concern a property of interest. Subgroup Discovery is a form of supervised inductive learning or subgroup description, in which the algorithm analyzes a set of data in order to find interesting subgroups, given a property of interest chosen by the user. The induction of rules that describe subgroups can be considered a multiobjective problem, since a Subgroup Discovery rule can be evaluated by means of different quality measures.

An application of an MOEA to Subgroup Discovery can be found in [120]. The algorithm, which is called the nondominated multiobjective evolutionary algorithm for extracting fuzzy rules in Subgroup Discovery (NMEEF-SD), is based on the well-known NSGA-II and aims to extract novel and interpretable fuzzy rules that describe subgroups. In NMEEF-SD, the quality measures that are considered as objectives in the evolutionary process can be selected, making it possible to study the combinations of measures that provide better results. Three quality measures are available: support, confidence, and unusualness, i.e., the weighted relative accuracy of a rule. These last measures attempt to obtain a good tradeoff between the generality, interest, and precision of results.

B. Summary of Multiobjective Evolutionary Fuzzy Systems Designed for Fuzzy Association Rule Mining

Table III contains all contributions that deal with MOEFSs that are designed for mining fuzzy association rules, presented in chronological order. As with Tables I and II, a description of this type of table is given in Section III-D, but the column describing the FRBS approach is no longer necessary. The remaining fields assume the meanings previously explained.

In most cases, the classical measures of data mining, support and confidence, are used as objectives. The application of MOEAs to extract fuzzy association rules is quite recent, beginning in 2006. Therefore, the majority of works exploit the second-generation MOEA.

VI. Open Problems and New Trends in Multiobjective Evolutionary Fuzzy Systems

In this section, some current trends in the field of MOEFSs will be presented, and some recent contributions related to them will be described. In addition, some issues will be highlighted in order to focus researchers' attention on new problems that arise when using MOEFSs in real-world applications.

One important issue concerns the fact that MOEAs have not been specifically designed for MOEFSs, in which a chromosome represents parts of an FRBS and consequently assumes a complex structure that can even comprise a combination of binary, integer, and real coding. Moreover, MOEFSs have to take into account test errors, which are not usually present in EMOO benchmarks. Because of this fact, existent MOEAs may not be suited to optimize FRBS structures, thus producing suboptimal solutions.

Considering this issue and the current state of the art of MOEFSs that are described in the previous sections, we try to highlight some problems related to MOEFSs that should be investigated. The following subjects will be stated as open problems and briefly described:

1) performance evaluation of MOEFSs;
2) reliable interpretability measures;
3) objective dimensionality;
4) scalability issues;
5) application to imbalanced datasets;
6) automatic selection of the most suitable solution;
7) integration of decision maker's preferences;
8) design MOEFSs to generate type-2 fuzzy systems.

A. Performance Evaluation of Multi-Objective Evolutionary Fuzzy Systems Approaches

Comparing different multiobjective optimization techniques is a difficult task, since the optimization result is a set of nondominated solutions rather than a single solution. Researchers generally agree on considering two informal criteria to assess the quality of a solution set: The distance of the approximated points from the true Pareto front should be minimized, and solutions should be equally distribute along the front. Additionally, the extent of the obtained nondominated front should be maximized.

In the literature, several performance measures have already been proposed to consider these criteria and to evaluate the search capacity of algorithms. The following measures are widely used: attainment surfaces, hypervolume, epsilon dominance [121]–[123], etc. A drawback of these measures is that the quality difference between the obtained FRBSs remains unclear. Moreover, the Pareto front approximation is generated with respect to the training data, whereas the performance of the algorithm should be evaluated with respect to test data by applying a statistical analysis.

A novel attempt based on the ideas in [43] to obtain representative mean values has been proposed in [65] to compare different multiobjective approaches: For each dataset and for each trial of an algorithm (considering cross validation), the approximated Pareto front is generated and three representative points are extracted (the best in the first objective, the median considering both objectives, and the best in the second objective). Afterward, for each dataset, the mean values and the standard deviations of some measures (first objective or training accuracy, second objective or complexity and test accuracy) are computed for each representative point over all the trials, and a nonparametric statistical test is applied locally for each measure at each representative point. This way, the authors were able to statistically compare the different algorithms by analyzing the performance of the obtained FRBSs when looking for the desired properties in the Pareto front extremes and in the midpoint (equilibrium point).

This approach has been extended in [29] and applied to problems with more than two objectives. To make a statistical comparison of the different interesting points possible, the authors project the obtained Pareto fronts on the planes generated by considering pairs of objectives (in this case, accuracy–complexity and accuracy–semantic planes). This way, they can analyze the nondominated solutions by considering the said interesting points for each pair of objectives.

This technique presents some problems when the Pareto fronts generated by different algorithms reside in distant zones of the objective space, as it is not applicable in these cases. Therefore, a previous graphical representation of the averaged Pareto fronts is necessary to determine whether this technique is suitable or not. In cases where the obtained Pareto fronts are located in different parts of the objective space, it could be determined which representative points are comparable for each dataset by considering this graphical representation, constituting a first attempt to assess the quality difference between fronts.

B. Reliable Interpretability Measures

In Section III, we explain how the definition of interpretability heavily affects the comprehensibility of an FRBS and researchers are still looking for reliable and widely accepted interpretability measures. Some proposals attempt to define new indices to consider multiple interpretability measures [28], [29], [124]. This problem is mainly related to contributions of the first category, and it is discussed deeply in [27], where a taxonomy is proposed to organize the different measures or constraints that are used in the literature to assess interpretability in linguistic FRBSs. A taxonomy with four quadrants is presented: complexity and semantic interpretability are taken into account at the level of RB or at the level of fuzzy partitions. Since the interpretability of linguistic FRBSs is still an open problem, the review tries to organize the different measures proposed so far, in order to help researchers to determine the most appropriate measures according to the part of the KB in which they want to maintain or improve interpretability.

This research [27] highlighted that there is not a single comprehensive global measure to quantify the interpretability of linguistic models; thus, it would be necessary to consider appropriate measures from all the quadrants. It is necessary to establish a way to combine these measures globally. To this end, the different measures might be optimized as different objectives within a multiobjective framework, by also taking accuracy into account. However, the real problem resides in the choice of common and widely accepted measures for each of the quadrants, which is still an open problem for the useful application of MOEFSs that aim to discover the accuracy–interpretability tradeoff of FRBSs.

C. Objective Dimensionality

MOEAs usually work very well for two or three objective problems, whereas their search capacity worsens as the number of objectives increases. Problems with four or more objectives are often called many-objective problems [125].

These kinds of problems can be handled by different approaches:

1) integrating many aspects into few objectives;
2) selecting few aspects as objectives;
3) using all the objectives.

The first approach aims to combine several objectives into a single one, using weights or appropriate aggregation operators. This method presents the common problems of a single-objective approach: the aggregation method and weights have to be chosen carefully, since they greatly influence the performance of the optimization process. However, it represents an effective way to handle many objectives when some of them are related and can be properly combined.

The second approach is achieved by reducing the dimensionality in the objective space, since not all the objectives may be necessary. If there is a certain number of nonconflicting objectives, these objectives must be considered redundant. On the other hand, in some cases there are some objectives (conflicting or not) that could be removed without significantly losing the problem information, in which case only the statistically significant conflicting objectives should be considered.

The third method is the most complex one, as when applying a classic MOEA to a many-objective problem, several problems arise. When the number of objectives increases, almost all solutions in a population become nondominated; therefore, the search capacity of MOEAs based on the Pareto-dominance concept is heavily affected. The number of solutions required to approximate the entire Pareto front increases exponentially with the number of objectives. This happens because in many-objective problems the Pareto front is represented by a hypersurface in the objective space. The decision making process becomes harder, since the final solution is chosen from among a wider number of multiobjective solutions.

To overcome these problems, researchers found that the low selection pressure could be tackled by inducing a preference ordering over the points in the nondominated set. The approaches that are based on preference ordering include relaxing the concept of Pareto dominance, controlling the dominance area, modifying the rank definition, substituting the distance metric, etc. These approaches seem promising, but they still need further investigation.

D. Scalability Issues

In recent years, having to deal with large or high-dimensional datasets has become more common [16], [126]. Large datasets include many instances, while high-dimensional datasets refer to datasets with a large number of features. These kinds of datasets provide some difficulties: the size of large datasets affects the fitness function computation, thus increasing the computational time, whereas high-dimensional datasets increase the search space. Moreover, in most of the cases, the wider the search space, the greater the number of generated rules. Resulting models can be very complex, with interpretability heavily affected. This problem is particularly evident in the works belonging to the first and third groups of the taxonomy.

In the case of large datasets, these problems can be tackled by reducing the training set, i.e., removing irrelevant training instances prior to the learning process. The choice of the subset is a crucial task, since it has to describe the whole training set without the loss of information. When dealing with high-dimensional datasets, it is also possible to perform a feature selection process that determines the most relevant variables before or during the learning process. Finally, the interpretability issue can be tackled by reducing the rule set through a postprocessing approach.

Large and high-dimensional datasets increasingly occur in real-word problems, but until now there have been few works that attempt to approach them through the multiobjective evolutionary optimization of fuzzy systems; therefore, this is still an interesting investigation field. A recent example can be found in [73], which proposes an MOEA to obtain linguistic Mamdani compact models in 17 regression problems, including up to 80 variables and up to 40 000 example data. A variable selection mechanism is applied to ensure a fast convergence in the presence of a high number of variables. To handle problems with a high number of examples, an error estimation of the obtained models is computed by using a reduced subset of the training patterns within a new mechanism for fitness estimation which is applicable to any EA.

E. Imbalanced Datasets

Problems with imbalanced datasets appear mainly when dealing with classification tasks [127]. Usually, the accuracy of a classifier is evaluated according to the percentage of correct classification, which should be maximized by the optimization process. This measure is inappropriate when the application domain is characterized by a highly imbalanced distribution of samples, since positive cases compose just a small fraction of the available data used to train the classifier. In some cases, the cost of misclassification is different between the positive and negative classes. Thus, the obtained classifier presents a high predictive accuracy over the majority class and poor predictive accuracy over the minority class. Furthermore, the minority class examples can be considered as noise and completely ignored.

Two approaches can be followed to reduce or avoid bias toward the majority class.

1) *At data level*: Preprocessing mechanisms can be applied to patterns to prevent imbalance. These solutions include different forms of resampling, i.e., oversampling, undersampling, and variations on or combinations of the previous techniques.
2) *At algorithmic level*: Solutions are mainly based on cost-sensitive approaches, by using metrics that take into account the misclassification costs of each class.

With regard to MOEFSs, imbalanced datasets could be handled in the application of FRBCs. The first approach in this sense can be found in [63], in which the performance of binary FRBCs is analyzed, considering an application domain characterized by highly imbalanced distributions of examples. To assess FRBCs' performance, two objectives are maximized: sensitivity and specificity. Sensitivity corresponds to the TP rate, while specificity corresponds to the complement of the FP rate. These two metrics describe the system's ability to correctly classify patterns belonging to both the positive and negative classes. The sum of the conditions in the antecedents of rules in the classifier is added as a third objective, in order to decrease the complexity. After the optimization process, the receiver operating characteristic (ROC) curve analysis is used to compare the obtained binary classifiers and to select a set of potentially optimal classifiers.

Since these kinds of datasets are increasingly used in several fields, such as security systems, medicine, telecommunication systems, information retrieval tasks, etc., they are receiving increasing attention from researchers.

F. Automatic Selection of the Most Suitable Solution

The strength of MOEAs resides in their ability to approximate a wide part of the Pareto front, thus providing multiple solutions with different tradeoffs between objectives. However, in many application fields, only a single solution is required. The problem of automatically choosing a single solution for a specific purpose has not been discussed in the studies presented so far.

Focusing on a set of obtained FRBSs (and on a single FRBS) represents a way to ease the choice of an appropriate single solution. However, this kind of visualization is a difficult task when the number of objectives increases, since it is impossible to show all the nondominated solutions in many-dimensional visualization spaces.

The obtained FRBSs also present the problem of overfitting since they are evaluated according to test data (generalization ability). Since MOEFSs are expected to obtain a large set of FRBSs, the choice of a single solution should consider FRBSs with good generalization abilities. However, this is not an easy task since it has to be included in the learning process; therefore, it is only possible to take into account the results of the training set, while the test set remains unused.

An approach to determine the most suitable FRBS from a given Pareto front in terms of its generalization ability has been proposed in [128]. In this contribution, the authors propose a technique using a double cross validation to evaluate the generalization ability of the obtained models. Double cross validation has a nested structure of two cross-validation loops. The inner loop is used to determine the best complexity of FRBSs with the highest generalization ability for the training data in each run in the outer loop. That is, the inner loop plays the role of validation data. The determined best complexity is used to choose the final FRBS in each run in the outer loop.

G. Integration of Decision Maker's Preferences

In a multiobjective optimization problem, exploring the whole search space can be unnecessary if the final goal is to find only those solutions that satisfy some requirements specified by the decision maker. A good strategy may be to direct the search process toward the areas of the Pareto front that better reflect the decision maker's preferences, by integrating these preferences into the optimization process. This way, the search space is reduced, and the efficiency of the search process is significantly increased. The incorporation of decision maker's preferences is an interesting research issue which has yet to be well explored in the literature.

In [129], the problem of multicriteria decision making (MCDM) is considered as the conjunction of three components: the search of the possible solutions, a preference tradeoff process to select a single solution, and an interactive visualization process to embed the decision maker in the solution refinement and selection loop. The authors introduce a requirement framework to compare most MCDM problems, compare their solutions, and analyze their performances.

The second example is presented in [130], where user preferences are incorporated into a rule selection process of FRBSs for pattern classification problems. Because of the difficulty in choosing an objective interpretability measure, multiple interpretability criteria are combined into a single *preference function*, which is used as one of the objective functions during the optimization process. Moreover, the preference function can be changed interactively by the user through the modification of the priority of each interpretability criterion.

Another possibility to indirectly consider user's preferences is to concentrate the search on the most significant objectives. Usually, when dealing with MOEFSs, the objectives used present different difficulty levels. This way, objectives that are easy to achieve, such as the complexity of the obtained models, bias the search, leading to suboptimal models (overly simple models presenting inappropriate accuracies when using complexity measures). However, the user is not only interested in obtaining simple models but also accurate ones. Some approaches concentrating the search on the accuracy objective as a way to obtain the most accurate models can be found in [41], [43], and [93].

H. Design Multiobjective Evolutionary Fuzzy Systems to Generate Type-2 Fuzzy Systems

At the end of the 1990s, a new class of fuzzy system was presented [131], in which the antecedent or consequent membership functions were type-2 fuzzy sets. The concept of a type-2 fuzzy set is introduced by Zadeh [132] as a generalization of the concept of an ordinary fuzzy set, also referred to as type-1 fuzzy set. A type-2 fuzzy set incorporates uncertainty about the membership function into fuzzy set theory since its membership function is 3-D, where the third dimension is the value of the membership function at each point on its 2-D domain. If there is no uncertainty, a type-2 fuzzy set is reduced to a type-1 fuzzy set. Such sets are useful when it is difficult to determine an exact membership function for a fuzzy set.

As in the case of type-1 fuzzy systems, the hybridization of type-2 fuzzy systems and GAs was proposed in [133], in order to automatically design type-2 fuzzy systems, following which several contributions have been published, in which GAs, and in general EAs, are used to obtain type-2 fuzzy systems, mainly in control applications [134]–[136].

Despite this, as far as we know, no proposals have yet been presented to combine MOEAs with type-2 fuzzy systems; therefore, this may be a new and promising research field.

VII. Conclusion

The application of MOEAs to fuzzy systems has received great attention from the research community for the past 15 years. MOEFSs can take into account multiple goals within the same optimization process, thus generating a set of nondominated fuzzy systems that represent a tradeoff among objectives. MOEFSs have been applied in several fields, due to their ability to represent real-world problems in a simple way and to include previous knowledge in the model. The number of contributions in this area has increased greatly in recent years. This paper has provided an overview of MOEFSs, suggesting an organization of the contributions in this area according to their types.

To this end, in this contribution we have proposed a two-level taxonomy, in which the first level is arranged depending on the multiobjective nature of the problem tackled and the second one on the type of GFS used.

The most prolific category includes works on the application of MOEFSs to the tradeoff between interpretability and accuracy. Therefore, many complex variations of existing MOEAs have been proposed in order to obtain better performances.

The second category concerns works that deal with the application of MOEFSs to multiobjective fuzzy control problems, in which many contributions focus on first-generation algorithms, probably due to the fact that they could be efficiently applied in control problems, in spite of their simplicity. However, it should be remembered that the introduction of the elitism concept in second-generation MOEAs is a theoretical requirement to assure convergence.

Only recently have MOEFSs been applied to extract fuzzy knowledge from databases; therefore, this category comprises few contributions. In addition, there are no well-described measures that consider fuzziness in association rules.

Finally, several current trends and open problems have been highlighted, in order to draw the attention of the research community to their importance, since they are either unsolved or have still not been addressed.

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